DUAL DIG LEVEL II 2008

- 1. Find the missing entry in the following infinite sequence: 1001, 100, _____, 14, 13, 12, 11, 10, 9, 9, 9, ...
- 2. Find one-to-one functions f and g with the following characteristics (a and b are numbers, a > 0 and b > 0):

a)
$$f(a + b) = f(a) \cdot f(b)$$

b)
$$g(a \cdot b) = g(a) + g(b)$$

3. Find the sum of the coefficients of all the terms after $(x - 2y + 3z)^3$ is expanded.

4. Calculate
$$\sum_{k=101}^{1000} k$$

5. Calculate
$$\sum_{i=1}^{3} \sum_{k=1}^{2} jk$$

6. Express -3i in polar form.

7. Find all the vertical asymptotes of the function:
$$f(x) = \frac{-x^3 - 2x^2 + 3x}{2x^4 - 2x^3 - 8x^2 + 8x}$$

8. Runners A, B, C, and D have chance of 0.3, 0.2, 0.1, and 0.4 respectively of winning a race. If A drops out of the race, what is the probability that B wins the race?

9. Solve:
$$x^4 - 2x^3 - 7x^2 + 18x - 18 = 0$$

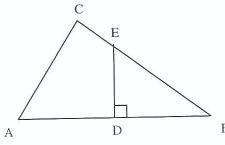
10. Find, if it exists, the horizontal asymptote of:
$$f(x) = \frac{x^2 + 3x + 10000}{2x^2 + x + 7}$$

11. A survey is taken on methods of commuter travel. Each person checks bus, train, or car as a method of traveling to work. More than one pick is permitted. The results are:

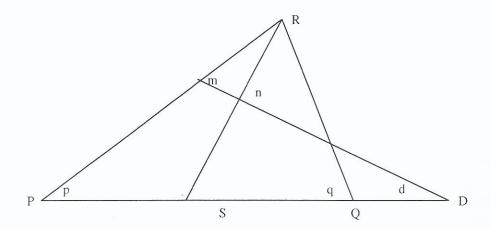
Bus	Train	Car	Bus & Train	Bus & Car	Train & Car	All three
30	35	100	15	15	20	5

How many people completed the survey?

- 12. Water is poured into a conical paper cup at the rate of 2/3 cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast does the water level rise when the water is 4 inches deep?
- 13. At noon a sailboat is 20 km south of a freighter. The sailboat is traveling east at 20 km per hour, and the freighter is traveling south at 40 km per hour. If visibility is 10 km, could the people on the two ships ever see each other?
- 14. Given 12 points in a plane no three of which are collinear, the number of lines they determine is:
- 15. The radioactive element carbon-14 has a half-life of 5750 years. Archaeologists discovered that the linen wrapping from one of the Dead Sea Scrolls had lost 22.3% of its carbon-14 at the time it was found. How old was the linen wrapping?
- 16. In the figure, it is given that angle $C = 90^{\circ}$, $\overline{AD} \cong \overline{DB}$, $\overline{DE} \perp \overline{AB}$, AB = 20, and AC = 12. The area of quadrilateral ADEC is:



- 17. Find real number constants P and Q such that $\frac{p}{x+1} + \frac{q}{x-3} = \frac{5x-3}{x^2 2x 3}$
- 18. Given triangle PQR with \overline{RS} bisecting angle R, \overline{PQ} extended to D and angle n a right angle, then define the angle m in terms of angles d, n, p, or q:



- 19. Find real numbers x and y such that 3x + 2iy ix + 5y = 7 + 5i
- 20. In how many ways can seven people be seated at a round table?